Chapter 24. Instrumental Variables Analysis of Randomized Experiments with Two-Sided Noncompliance

이종진

Seoul National University

ga0408@snu.ac.kr

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Previous chapters...

- Unconfoundness of the treatment of interest is questionable
- With the existence of noncompliers (one-sided)
  - (W_i) / confounded
  - Instrumental variable $\rightarrow$ estimate "local" average effects for the subpopulation
- Completely randomized design and one assumption
  - Completely randomized $\rightarrow$ (Z_i), unconfoundness $\rightarrow$ enable to esitmate ITT.
  - Exclusion assumptions $\rightarrow$ allow to estimate "local" average effects for the compliers

24.1 Introduction

- IV analyses for two-sided noncompliance in a randomized experiment.
- Completely randomized and two assumptions
  - Completely randomized $\rightarrow$ (Z_i), unconfoundness $\rightarrow$ ITT.
  - Exclusion assumptions and monotonicity assumption $\rightarrow$ allow to estimate "local" average effects for the subpopulation of compliers

24.2 The Angrist Draft Lottery Data

- **Serving in the military (veteran / non-veteran) → earning**
- Angrist(1990) exploits the implementation of the draft during the Vietnam War.
  - All men of a certain age were required to register for the draft.
  - Draft priority was assigned randomly based on the birth dates per birth year
- Possibility of the existence of complier
  - Medical test / minimum educational level
  - Low lottery number → decided to enter graduate school / move to Canada

The Angrist Draft Lottery Data

Table 24.1. *Summary Statistics for the Angrist Draft Lottery Data*

	Non-Veterans ($N_c = 6{,}675$)				Veterans ($N_t = 2{,}030$)			
	Min	Max	Mean	(S.D.)	Min	Max	Mean	(S.D.)
Draft eligible	0	1	0.24	(0.43)	0	1	0.40	(0.49)
Yearly earnings (in $1,000's)	0	62.8	11.8	(11.5)	0	50.7	11.7	(11.8)
Earnings positive	0	1	0.88	(0.32)	0	1	0.91	(0.29)
Year of birth	50	52	51.1	(0.8)	50	52	50.9	(0.8)

24.3 Compliance Status

- Compliance Status
  - A function of the pair of potential responses $(W_i(0), W_i(1))$
- Compliance types
  - Denote compliance type by G_i

$$G_i = g\left(W_i(0), W_i(1)\right) = \begin{cases} \text{nt } (\textit{nevertaker}) & \text{if } W_i(0) = 0, W_i(1) = 0, \\ \text{co } (\textit{complier}) & \text{if } W_i(0) = 0, W_i(1) = 1, \\ \text{df } (\textit{defier}) & \text{if } W_i(0) = 1, W_i(1) = 0, \\ \text{at } (\textit{alwaystaker}) & \text{if } W_i(0) = 1, W_i(1) = 1. \end{cases}$$

Compliance Status-One-sided case

- We only observed the realized treatment status of $W_i(0)$ or $W_i(1)$
- One-sided
  - If $Z_i = 1, W_i^{obs} = 0 \rightarrow (W_i(0), W_i(1)) = (0, 0)$ / nc
  - If $Z_i = 1, W_i^{obs} = 1 \rightarrow (W_i(0), W_i(1)) = (0, 1)$ / co
  - If $Z_i = 0, W_i^{obs} = 0 \rightarrow (W_i(0), W_i(1)) = (0, 0)$ or $(0, 1)$ / co or nc
  - If $Z_i = 0, W_i^{obs} = 1 \rightarrow (W_i(0), W_i(1)) = (1, 0)$ or $(1, 1)$ / co or nc

Compliance Status-One-sided case

- Two-sided
  - If $Z_i = 1, W_i^{obs} = 0 \rightarrow (W_i(0), W_i(1)) = (0,0)$ or $(1,0)$ / nt or df
  - If $Z_i = 1, W_i^{obs} = 1 \rightarrow (W_i(0), W_i(1)) = (0,1)$ or $(1,1)$ / df or at
  - If $Z_i = 0, W_i^{obs} = 0 \rightarrow (W_i(0), W_i(1)) = (0,0)$ or $(0,1)$ / nt or co
  - If $Z_i = 0, W_i^{obs} = 1 \rightarrow (W_i(0), W_i(1)) = (1,0)$ or $(1,1)$ / df or at
- That is why two-sided noncompliance case is more comlicated.
  - Need additional assumption to identify the causal effect
  - Monotonicity $\rightarrow$ No defier

24.4 Intention-To-Treat Effects

- Largely unchanged from the one-sided case
- Unit-level effect of 4 compliance types
  - 1 for co
  - 0 for nt and at
  - -1 for df
- Super-population average ITT

$$\mathrm{ITT_W} = \mathbb{E}_{\mathrm{sp}}\left[W_i(1) - W_i(0)\right] = \pi_{\mathrm{co}} - \pi_{\mathrm{df}}$$

- The ITT effect on the primary outcome

$$\mathrm{ITT_Y} = \mathbb{E}_{\mathrm{sp}}\left[Y_i\left(1, W_i(1)\right) - Y_i\left(0, W_i(0)\right)\right]$$

Random Assignment of Z_i

- **Assumption 24.1 (Super-Population Random Assignment)**

$$Z_i \perp (W_i(0), W_i(1), Y_i(0,0), Y_i(0,1), Y_i(1,0), Y_i(1,1))$$

ITT Estimands for W

- The average causal effect of assignment on W_i

$$\widehat{\mathrm{ITT}_{\mathrm{W}}} = \bar{W}_1^{\mathrm{obs}} - \bar{W}_0^{\mathrm{obs}}$$

where $z = 0, 1$, $N_z = \sum_{i=1}^{N} 1_{Z_i = z}$, $\bar{W}_z^{\mathrm{obs}} = \sum_{i:Z_i=z} W_i^{\mathbf{obs}} \, / N_z$

- with (Neyman) sampling variance estimated as

$$\widehat{\mathbb{V}}(\widehat{\mathrm{ITT}_{\mathrm{W}}}) = \frac{s_{Y,1}^2}{N_1} + \frac{s_{Y,0}^2}{N_0}$$

where

$$s_{W,z}^2 = \sum_{i:Z_i=z} \left(W_i^{\mathrm{obs}} - \bar{W}_z^{\mathrm{obs}}\right)^2 / \left(N_z - 1\right) = \bar{W}_z \left(1 - \bar{W}_z\right) / \left(N_z - 1\right).$$

ITT Estimands for Y

- The difference in average outcomes by assignment status,

$$\widehat{\mathrm{ITT}_{\mathrm{Y}}} = \bar{Y}_1^{\mathrm{obs}} - \bar{Y}_0^{\mathrm{obs}}$$

where $z = 0, 1$, $N_z = \sum_{i=1}^{N} 1_{Z_i = z}$, $\bar{Y}_z^{\mathrm{obs}} = \sum_{i:Z_i=z} Y_i^{\mathbf{obs}} \,/\, N_z$

- with (Neyman) sampling variance estimated as

$$\widehat{\mathbb{V}}(\widehat{\mathrm{ITT}}) = \frac{s_{W,0}^2}{N_0} + \frac{s_{W,1}^2}{N_1}$$

where $s_{Y,z}^2 = \sum_{i:Z_i=z} \left(Y_i^{\mathrm{obs}} - \bar{Y}_z^{\mathrm{obs}}\right)^2 / \left(N_z - 1\right)$

24.5 Instrumental Variables

- Main resluts of this chapter
- Consider assumptions underlying instrument variables to draw inferences about the relation W_i and Y_i
  - Exclusion Retrictions
  - Monotonicity Assumption

Exclusion Restrictions

- **Assumption 24.2 (Exclusion Restriction for Nevertakers)** For all units i with $G_i = \text{nt}$,
$$Y_i(0,0) = Y_i(1,0)$$
- **Assumption 24.3 (Exclusion Restriction for Alwaystakers)** For all units i with $G_i = \text{at}$,
$$Y_i(0,1) = Y_i(1,1)$$
- **Assumption 24.4 (Exclusion Restriction for Compliers)** For all units i with $G_i = \text{co}$,
$$Y_i(0,w) = Y_i(1,w)$$
for both levels of the treatment w
- **Assumption 24.5 (Exclusion Restriction for Defiers)** For all units i with $G_i = \text{df}$,
$$Y_i(0,w) = Y_i(1,w)$$
for both levels of the treatment w

ITT Estimands for Y

$$
\begin{aligned}
\mathrm{ITT_Y} =& \mathbb{E}_{\mathrm{sp}}[Y(1, W(1)) - Y(0, W(0))] \\
=& \sum_{g \in \{\mathrm{co,nt,at,df}\}} \mathbb{E}_{\mathrm{sp}}\left[Y_i\left(1, W_i(1)\right) - Y_i\left(0, W_i(0)\right) \mid G_i = g\right] \cdot \Pr_{\mathrm{sp}}\left(G_i = g\right) \\
=& \mathbb{E}_{\mathrm{sp}}\left[Y_i\left(1, W_i(1)\right) - Y_i\left(0, W_i(0)\right) \mid G_i = \mathrm{co}\right] \cdot \Pr_{\mathrm{sp}}\left(G_i = \mathrm{co}\right) \\
&+ \mathbb{E}_{\mathrm{sp}}\left[Y_i\left(1, W_i(1)\right) - Y_i\left(0, W_i(0)\right) \mid G_i = \mathrm{nt}\right] \cdot \Pr_{\mathrm{sp}}\left(G_i = \mathrm{nt}\right) \\
&+ \mathbb{E}_{\mathrm{sp}}\left[Y_i\left(1, W_i(1)\right) - Y_i\left(0, W_i(0)\right) \mid G_i = \mathrm{at}\right] \cdot \Pr_{\mathrm{sp}}\left(G_i = \mathrm{at}\right) \\
&+ \mathbb{E}_{\mathrm{sp}}\left[Y_i\left(1, W_i(1)\right) - Y_i\left(0, W_i(0)\right) \mid G_i = \mathrm{df}\right] \cdot \Pr_{\mathrm{sp}}\left(G_i = \mathrm{df}\right) \\
=& \mathbb{E}_{\mathrm{sp}}\left[Y_i(1,1) - Y_i(0,0) \mid G_i = \mathrm{co}\right] \cdot \pi_{\mathrm{co}} \\
&- \mathbb{E}_{\mathrm{sp}}\left[Y_i(0,1) - Y_i(1,0) \mid G_i = \mathrm{df}\right] \cdot \pi_{\mathrm{df}}
\end{aligned}
$$

- **Assumption 24.8 (Monotonicity/No Defiers)**

$$W_i(1) \geq W_i(0)$$

- With assumptions 24.4 and 24.8

$$\mathrm{ITT}_{\mathrm{Y}} = \mathbb{E}_{\mathrm{sp}}\left[Y_i(1) - Y_i(0) \mid G_i = \mathrm{co}\right] \cdot \pi_{\mathrm{co}}$$

- **Theorem 24.1 (Local Average Treatment Effect)**

  with Assumptions 24.1-24.4 and 24.8 hold.

$$\tau_{\mathsf{late}} = \frac{\mathrm{ITT}_{\mathrm{Y}}}{\mathrm{ITT}_{\mathrm{W}}} = \mathbb{E}_{\mathrm{SP}}\left[Y_i(1) - Y_i(0) \mid G_i = \mathrm{co}\right]$$

Conclusion

- Introduce types of noncompliance
- Extend to two-sided noncompliance cases
  - Completely randomized experiments
  - Exclusion restrictions and monotonicity assumption

The End